Large Deflection Microstructure Theory for a Composite Beam

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A continuum model with microstructure is developed for a composite beam under large deflection. The von Kármán strain-displacement relations are employed. The assumed approximate displacement field accounts for the gross transverse shear deformation and the local transverse shear deformation in the constituents. The equations of motion, as well as the boundary conditions, are derived. The theory is employed to investigate the influence of large amplitudes on flexural wave propagation and free vibrations. It is found that the anisotropy of the composite beam substantially affects the flexural motion through the geometric nonlinearity incorporated in the model.

Introduction

THE increasing use of structural composite materials has inspired the intensive study of heterogeneous solids. The conventional approach in analyzing the mechanical response of composite media has been the effective modulus theory through which the composites are treated as classical simple solids. Recent investigations have revealed the practical significance of the microstructure theories, such as the one developed by Mindlin.[1] Sun, Achenbach and Herrmann[2] have shown in describing the dynamic response of a laminated medium that the continuum model with microstructure is a better one. A linear microstructure theory for a composite beam was subsequently developed by Sun,[3] who found good agreement between the theory and the exact analysis in the case of flexural wave propagation. It was also shown in Ref. 3 that the absence of microstructure could result in serious discrepancies even in the range of rather long wavelengths.

In the present paper, we are concerned with the development of a microstructure theory for a composite beam in large-amplitude motion. The equations of motion, as well as the boundary conditions, are derived by the energy principle. The model thus established for the composite beam allows both gross shear deformation and individual shear deformations in the constituent layers. The equations are used to investigate the influence of large amplitudes on flexural wave propagation in, and free vibrations of, the composite beam. Numerical examples are given and discussed.

Derivation of the Governing Equations

The composite beam is assumed to consist of a large number of layers of two different materials which are each homogeneous and orthotropic, with the material planes of symmetry coinciding with the coordinate planes. The two types of layers, denoted by materials "1" and "2", respectively, are stacked alternately (see Fig. 1). Each layer of material (1) has thickness d_1, while that of material (2) has thickness d_2. Without loss of generality, we assume that material (1) is stiffer than material (2).

We first focus our attention on the kth pair of layers. An approximate expression of the displacement in the layers which includes transverse shear deformation is given by

$$\bar{u}_\alpha^k = u_\alpha^k(x,t) - \bar{y}_\alpha^k \phi_\alpha^k(x,t)$$
$$\bar{w}_\alpha^k = w_\alpha^k(x,t), \quad \alpha = 1,2 \tag{1}$$

where $\bar{u}_\alpha^k$ and $\bar{w}_\alpha^k$ denote, respectively, the extensional and transverse displacements in the kth layer of material $1(\alpha = 1)$ and material $2(\alpha = 2)$; u_α^k and w_α^k are the displacements at the midplanes, ϕ_α^k indicates the rotations of the cross sections, and $\bar{y}_\alpha^k$ the local thickness coordinates in the layers (Fig. 1).

We assume that the composite beam undergoes large deflections for which the following von Kármán-type of strain-displacement relations is taken in each constituent layer

$$\varepsilon_{xx\alpha}^k = \partial\bar{u}_\alpha^k/\partial x + \tfrac{1}{2}(\partial\bar{w}_\alpha^k/\partial x)^2$$
$$\varepsilon_{xy\alpha}^k = \tfrac{1}{2}[\partial\bar{w}_\alpha^k/\partial x + \partial\bar{u}_\alpha^k/\partial\bar{y}_\alpha^k], \quad \alpha = 1,2 \tag{2}$$

The pertinent Kirchhoff stress component $\sigma_{xx\alpha}^k$ and $\sigma_{xy\alpha}^k$ are assumed to be linearly related to the strains as

$$\sigma_{xx\alpha}^k = E_\alpha \varepsilon_{xx\alpha}^k, \quad \sigma_{xy\alpha}^k = 2G_\alpha \varepsilon_{xy\alpha}^k \tag{3}$$

The strain energies per unit length in the layers are now obtained by integrating the strain energy density over the thickness of the respective layer. In terms of the approximate displacement given by Eq. (1), we write the strain energy per unit length of the layer as

$$U_\alpha^k = \frac{1}{2}E_\alpha A_\alpha\left[\frac{\partial u_\alpha^k}{\partial x} + \frac{1}{2}\left(\frac{\partial w_\alpha^k}{\partial x}\right)^2\right]^2 + \frac{1}{2}E_\alpha I_\alpha\left(\frac{\partial \phi_\alpha^k}{\partial x}\right)^2 + \frac{1}{2}G_\alpha A_\alpha\left(\frac{\partial w_\alpha^k}{\partial x} - \phi_\alpha^k\right)^2 \tag{4}$$

in which A_α and I_α are, respectively, the cross-sectional areas and the moments of inertia for the kth stiff layer ($\alpha = 1$) and the kth soft layer ($\alpha = 2$).

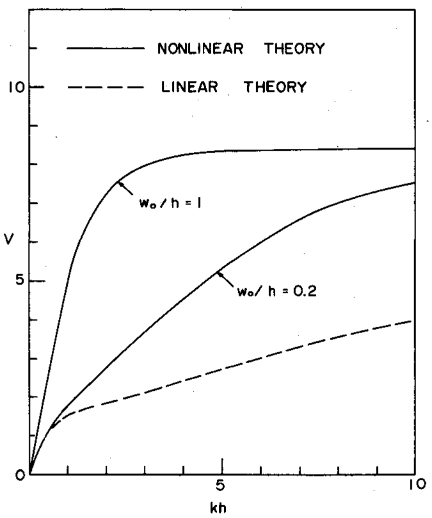

Fig. 1 The composite beam.

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The kinetic energies per unit length of the kth stiff and soft layers are

$$T_\alpha^k = \tfrac{1}{2}\rho_\alpha A_\alpha(\partial w_\alpha^k/\partial t)^2 + \tfrac{1}{2}\rho_\alpha I_\alpha(\partial \phi_\alpha^k/\partial t)^2 + \tfrac{1}{2}\rho_\alpha A_\alpha(\partial u_\alpha^k/\partial t) \quad (5)$$

where ρ_1 and ρ_2 are the mass densities for the stiff layer and soft layer, respectively.

Because of heterogeneity, deformation in the composite beam is obviously different from that of a homogeneous beam. The distinct material properties of the constituent layers yield distinct rotations (microrotations) in layers of different materials. Consequently, the assumption that a plane section remains plane after deformation is not valid. Instead, we may assume that the average displacements of the individual layers, i.e., the displacement of the midplanes of the layers, remain in a (fictitious) plane after deformation. The rotations of the constituent layers are then allowed to deviate from the plane to account for the heterogeneous characteristics of the composite beam. With the foregoing in mind we write the displacements in the midplanes of the kth pair of layers as

$$u_\alpha^k = u(x,t) - y_\alpha^k\psi(x,t)$$
$$w_\alpha^k = w(x,t), \quad \alpha = 1,2 \quad (6)$$

where u is the extensional displacement of the midplane of the composite beam, w is the transverse displacement, ψ indicates the gross rotation of the (fictitious) plane, and y_α^k denotes the positions of the midplanes of the layers. The assumption given by Eq. (6) has been shown to be quite satisfactory in the linear case.[3]

With the use of Eq. (6), the strain and kinetic energies given by Eqs. (4) and (5), respectively, can be written in terms of the new kinematic variables u, w, ψ, ϕ_α^k and y_α^k.

The strain energy per unit length of the composite beam can be obtained by summing up the strain energies stored in the individual layers. Such quantity is now expressed in terms of of the continuous functions $u(x,t)$, $\psi(x,t)$, $w(x,t)$ and the discrete variables ϕ_α^k and y_α^k. However, by the smoothing operation[3] through which the summation is replaced by an integral over the thickness of the beam, consequently smoothing out the discrete variables, the strain energy per unit length of the composite beam can be expressed in the form

$$U = \frac{1}{2}\xi(E_1A_1 + E_2A_2)\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]^2 + \frac{1}{2}I_b[\eta E_1 + (1-\eta)E_2]\left(\frac{\partial \psi}{\partial x}\right)^2 + \frac{1}{2}\xi G_1A_1\left(\frac{\partial w}{\partial x} - \phi\right)^2 + \frac{1}{2}\xi E_1I_1\left(\frac{\partial \phi}{\partial x}\right)^2 + \frac{1}{2}\xi G_2A_2\left[\frac{\partial w}{\partial x} - \frac{1}{1-\eta}\psi + \frac{\eta}{1-\eta}\phi\right]^2 + \frac{1}{2}\frac{\xi}{(1-\eta)^2}E_2I_2\left[\frac{\partial \psi}{\partial x} - \eta\frac{\partial \phi}{\partial x}\right]^2 \quad (7)$$

where $\phi = \phi_1^k$ (all k), and represents the rotations in the stiff layers which have been assumed constant through the thickness of the beam; ξ and η are defined as

$$\xi = h/(d_1 + d_2), \quad \eta = d_1/(d_1 + d_2) \quad (8)$$

It must be mentioned that in deriving Eq. (7), the condition

$$\psi = \eta\phi_1^k + (1-\eta)\phi_2^k \quad (9)$$

which ensures the continuity of displacements at the interfaces of the layers has been used to eliminate ϕ_2^k [See Ref. 3 for detailed derivation of Eq. (9).].

The kinetic energy per unit length of beam is derived in the same manner. We have

$$T = \frac{1}{2}\xi(\rho_1A_1 + \rho_2A_2)\left(\frac{\partial u}{\partial t}\right)^2 + \frac{1}{2}\xi(\rho_1A_1 + \rho_2A_2)\left(\frac{\partial w}{\partial t}\right)^2 + \frac{1}{2}\xi\rho_1I_1\left(\frac{\partial \phi}{\partial t}\right)^2 + \frac{1}{2}\frac{\xi}{(1-\eta)^2}\rho_2I_2\left(\frac{\partial \psi}{\partial t} - \eta\frac{\partial \phi}{\partial t}\right)^2 + \frac{1}{2}I_b[\eta\rho_1 + (1-\eta)\rho_2]\left(\frac{\partial \psi}{\partial t}\right)^2 \quad (10)$$

Suppose that the composite beam is subjected to a distributed load $p(x,t)$ over the span of the beam, and at the ends the axial force N, the shear force Q, the gross moment M, and the micromoment m are acting. The variation of work done by the external forces for the variations δu, δw, $\delta\psi$ and $\delta\phi$ can be expressed in the form

$$\delta W_e = \int_0^l p\,\delta w\,dx + N(l)\,\delta u(l) - N(0)\,\delta u(0) + Q(l)\,\delta w(l) - Q(0)\,\delta w(0) + M(l)\,\delta\psi(l) - M(0)\,\delta\psi(0) + m(l)\,\delta\phi(l) - m(0)\,\delta\phi(0) \quad (11)$$

where l is the length of the beam. The expression given by Eq. (11) can be related to the actual forces and moments acting in the individual layers.[3]

The equations of motion and boundary conditions are obtained by invoking Hamilton's principle

$$\delta\int_{t_0}^{t_1}\int_0^l (T-U)\,dx\,dt + \int_{t_0}^{t_1}\delta W_e\,dt = 0 \quad (12)$$

The equations of motion are:

$$a_1[\partial^2u/\partial x^2 + (\partial w/\partial x)(\partial^2 w/\partial x^2)] = b_1(\partial^2 u/\partial t^2) \quad (13)$$

$$a_1\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\right) + a_2\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2} + a_3\frac{\partial^2 w}{\partial x^2} - a_4\frac{\partial \psi}{\partial x} - a_5\frac{\partial \phi}{\partial x} + p = b_1\ddot{w} \quad (14)$$

$$a_4(\partial w/\partial x) + a_6(\partial^2\psi/\partial x^2) - a_7\psi - a_8(\partial^2\phi/\partial x^2) + a_9\phi = b_2\ddot{\psi} - b_3\ddot{\phi} \quad (15)$$

$$a_5(\partial w/\partial x) - a_8(\partial^2\psi/\partial x^2) + a_9\psi + a_{10}(\partial^2\phi/\partial x^2) - a_{11}\phi = -b_3\ddot{\psi} + b_4\ddot{\phi} \quad (16)$$

where

$$a_1 = \xi(E_1A_1 + E_2A_2), \quad a_2 = \tfrac{3}{2}a_1$$
$$a_3 = \xi(G_1A_1 + G_2A_2), \quad a_4 = [1/(1-\eta)]\xi G_2A_2$$
$$a_5 = \xi\{G_1A_1 - [\eta/1-\eta)]G_2A_2\}$$
$$a_6 = I_b[\eta E_1 + (1-\eta)E_2] + [1/(1-\eta)^2]\xi E_2I_2 \quad (17)$$
$$a_7 = [\xi/(1-\eta)^2]G_2A_2, \quad a_8 = [\eta/(1-\eta)^2]\xi E_2I_2$$
$$a_9 = [\eta/(1-\eta)^2]\xi G_2A_2, \quad a_{10} = \xi[E_1I_1 + [\eta^2/(1-\eta)^2]E_2I_2]$$
$$a_{11} = \xi G_1A_1 + [\eta^2/(1-\eta)^2]\xi G_2A_2, \quad b_1 = \xi[\eta\rho_1 + (1-\eta)\rho_2]$$
$$b_2 = I_b(\eta\rho_1 + (1-\eta)\rho_2] + [1/(1-\eta)^2]\xi\rho_2I_2$$
$$b_3 = [\eta/(1-\eta)^2]\xi\rho_2I_2, \quad b_4 = \{\rho_1I_1 + [\eta^2/(1-\eta)^2]\rho_2I_2\}$$

The boundary conditions are:

$$a_1[\partial u/\partial x + \tfrac{1}{2}(\partial w/\partial x)^2] = N \quad (18)$$

$$a_1(\partial w/\partial x)\{\partial u/\partial x + \tfrac{1}{2}[\partial w/\partial x]^2\} + a_3(\partial w/\partial x) - a_4\psi - a_5\phi = Q \quad (19)$$

$$a_6(\partial\psi/\partial w) - a_8(\partial\phi/\partial x) = M \quad (20)$$

$$a_{10}(\partial\phi/\partial x) - a_8(\partial\psi/\partial x) = m \quad (21)$$

at $x = 0$ and $x = l$.

Propagation of Free Waves

Consider a composite beam of infinite extent subjected to no lateral loads, i.e., $p = 0$. As an application of the beam theory derived previously, the problem of propagation of free waves in the infinite beam is first investigated. Consider the harmonic waves of the following form:

$$u = u_0 \sin\bar{k}(x - \bar{c}t) \quad (22)$$
$$w = w_0 \cos k(x - ct) \quad (23)$$
$$\psi = \psi_0 \sin k(x - ct) \quad (24)$$
$$\phi = \phi_0 \sin k(x - ct) \quad (25)$$

where u_0, w_0, ϕ_0 and ϕ_0 are the constant amplitudes, $\bar{k}$ and k the wave numbers, and $\bar{c}$ and c the phase velocities. Equation (22) represents the train of longitudinal wave, while the flexural wave is described by Eqs. (23–25). The two trains of waves are coupled as a result of the nonlinearity of the equations of motion.

The relations between k, c and k, c can be obtained from a

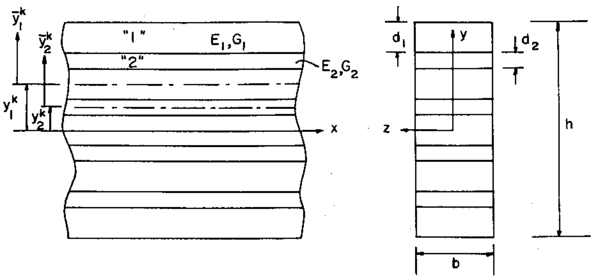

Fig. 2 Influence of large amplitude on propagation of free waves.

substitution of Eqs. (22) and (23) in Eq. (13). We obtain

$$\bar{k} = 2k, \quad \bar{c} = c \tag{26}$$

Substituting Eqs. (22–25) in Eqs. (13–16), we obtain

$$u_0 - [k/8(1 - c^2/c_p^2)]w_0^2 = 0 \tag{27}$$

$$(-a_1k^3w_0u_0 + \tfrac{1}{4}a_2k^4w_0^3 + a_3k^2w_0 - b_1k^2c^2w_0 + a_4k\psi_0 + a_5k\varphi_0)\cos k(x - ct) + (3a_1k^3w_0u_0 - \tfrac{1}{4}a_2k^4w_0^3)\cos 3k(x - ct) = 0 \tag{28}$$

$$-a_4kw_0 + (b_2k^2c^2 - a_6k^2 - a_7)\psi_0 + (-b_3k^2c^2 + a_8k^2 + a_9)\phi_0 = 0 \tag{29}$$

$$-a_5kw_0 + (-b_3k^2c^2 + a_8k^2 + a_9)\psi_0 + (b_4k^2c^2 - a_{10}k^2 - a_{11})\phi_0 = 0 \tag{30}$$

where

$$c_p^2 = (E_1A_1 + E_2A_2)/(\rho_1A_1 + \rho_2A_2) \tag{31}$$

It is noted that in Eq. (28), the fundamental mode and a higher mode are coupled. However, for small nonlinearities, the coupling effect of the higher mode should be also small, and we may neglect the second term in Eq. (28) to obtain

$$-a_1k^2w_0u_0 + \tfrac{1}{4}a_2k^3w_0^3 + (a_3 - b_1c^2)kw_0 + a_4\psi_0 + a_5\phi_0 = 0 \tag{32}$$

Using Eqs. (27), (32), (29), and (30), the three constants u_0, ψ_0 and φ_0 can be eliminated, and the dispersion equation can be thus obtained. The dispersion equation can be expressed in the nondimensional form as

$$(A_1A_2 + A_3)(w^*)^2 + A_4 + A_5B_1 + A_6B_2 = 0 \tag{33}$$

where

$$w^* = w_0/h \tag{34}$$

and

$$\begin{aligned}
&A_1 = L/8(1 - c^2/c_p^2), \quad A_2 = -a_1L^2/G_2h^2\\
&A_3 = a_2L^3/4G_2h^2, \quad A_4 = (a_3 - V^2b_1G_2/\rho_2)L/G_2h^2\\
&A_5 = a_4/G_2h^2, \quad A_6 = a_5/G_2h^2\\
&A_7 = -a_4L/G_2h^2, \quad A_8 = (b_2L^2V^2G_2/\rho_2h^2 - a_6L^2/h^2 - a_7)/G_2h^2\\
&A_9 = (-b_3L^2V^2G_2/\rho_2h^2 + a_8L^2/h^2 + a_9)/G_2h^2\\
&A_{10} = -A_6L, \quad A_{11} = A_9\\
&A_{12} = (b_4L^2V^2G_2/\rho_2h^2 - a_{10}L^2/h^2 - a_{11})/G_2h^2\\
&B_1 = A_7A_{12} - A_9A_{10}/A_{11}A_9 - A_8A_{12}\\
&B_2 = A_7A_{11} - A_8A_{10}/A_8A_{12} - A_9A_{11}
\end{aligned} \tag{35}$$

In Eq. (35), we have introduced the dimensionless wave number L and the dimensionless phase velocity V, defined as

$$L = kh, \quad V = c/(G_2/\rho_2)^{1/2} \tag{36}$$

The dispersion equation (33) can be solved for V for given values of w^* and L. In Fig. 2, the dimensionless phase velocity V is plotted against the dimensionless wave number L for $w^*(= w_0/h) = 0.2$ and 1, in conjunction with the following numerical values:

$$G_1/G_2 = 100, \quad \rho_1/\rho_2 = 2, \quad E_1/G_2 = 240, \quad E_2/G_2 = 2.7, \quad \eta = 0.8, \quad \xi = 4.8 \tag{37}$$

The dispersion curves obtained from the linear theory $w^* = 0$ are also shown in Fig. 2 for the sake of comparison. It can be seen easily that the coupling between the longitudinal and flexural waves is quite pronounced. The phase velocity of the flexural wave obtained based on the nonlinear theory is elevated greatly by the longitudinal wave through the nonlinearity. To a much more moderate extent than observed here, this coupling effect also exists in large-amplitude motions of a homogeneous isotropic plate.[4]

Large-Amplitude Vibration

The second application of the theory is concerned with the free vibration in large amplitudes of a finite composite beam of length l hinged at both ends. The solution must satisfy the following boundary conditions:

$$u = 0, \quad w = 0, \quad a_6\,\partial\psi/\partial x - a_8\,\partial\phi/\partial x = M = 0, \quad a_{10}\,\partial\phi/\partial x - a_8\,\partial\psi/\partial x = m = 0 \tag{38}$$

at $x = 0, l$.

For transverse vibration, the longitudinal inertia in Eq. (13) may prove to be of a higher order if the perturbation method is employed. This was shown by Chu and Herrmann[5] for an elastic homogeneous plate. For a preliminary investigation of the influence of large amplitudes on the vibration of the composite beam, we will neglect the inertia term associated with the longitudinal motion and the rotatory inertia terms in Eqs. (13), (15) and (16). This can be accomplished by dropping the term on the right-hand side of Eq. (13), in conjunction with setting $b_2 = b_3 = b_4 = 0$ in Eqs. (15) and (16).

With the foregoing simplification, we now seek the solution of the equations of motion. We assume the transverse displacement w to be of the form

$$w = w_0R(t)\sin(\pi x/l) \tag{39}$$

where w_0 is the amplitude and $R(t)$ is the time function. We also choose to measure the time such that

$$R = 1, \quad dR/dt = 0 \tag{40}$$

at $t = 0$.

Substituting Eq. (39) in Eqs. (13), (15), and (16) with the longitudinal and rotatory inertia terms being neglected, we obtain

$$u = -(w_0^2\pi R^2/8l)\sin 2\pi x/l \tag{41}$$

$$\psi = (w_0\pi R/l)\psi^*\cos\pi x/l \tag{42}$$

$$\phi = (w_0\pi R/l)\Phi^*\cos\pi x/l \tag{43}$$

in which

$$\psi^* = \begin{vmatrix} a_4 & -a_8\pi^2/l^2 - a_9 \\ a_5 & a_{10}\pi^2/l^2 + a_{11} \end{vmatrix} \Big/ \Delta \tag{44}$$

$$\Phi^* = \begin{vmatrix} a_6\pi^2/l^2 + a_7 & a_4 \\ -a_8\pi^2/l^2 - a_9 & a_5 \end{vmatrix} \Big/ \Delta \tag{45}$$

where

$$\Delta = \begin{vmatrix} a_6\pi^2/l^2 + a_7 & -a_8\pi^2/l^2 - a_9 \\ -a_8\pi^2/l^2 - a_9 & a_{10}\pi^2/l^2 + a_{11} \end{vmatrix} \tag{46}$$

It is noted that the general solutions given by Eqs. (39) and (41–43) satisfy the boundary conditions in Eq. (38).

Substitution of Eqs. (39) and (41–43) in Eq. (14) with $p = 0$ yields

$$\{b_1(d^2R/dt^2) + (\pi^2/l^2)[a_3 - a_4\psi^* - a_5\Phi^*]R + (\pi^4/8l^4)[2a_2 - a_1]w_0^2R^3\}\sin\pi x/l = 0 \tag{47}$$

The equation governing the time function $R(t)$ is obtained from Eq. (47) as

$$b_1(d^2R/dt^2) + (\pi^2/l^2)[a_3 - a_4\psi^* - a_5\Phi^*]R + (\pi^4/8l^4)[2a_2 - a_1]w_0^2R^3 = 0 \tag{48}$$

together with the initial conditions given by Eq. (40).

The solution for the nonlinear ordinary differential equation (48) is readily observed to be an elliptic function.[6] Using the conditions given by Eq. (40), the solution can be written in terms of an elliptic cosine function, cn, as

$$R(t) = \mathrm{cn}(\omega^*t, \beta) \tag{49}$$

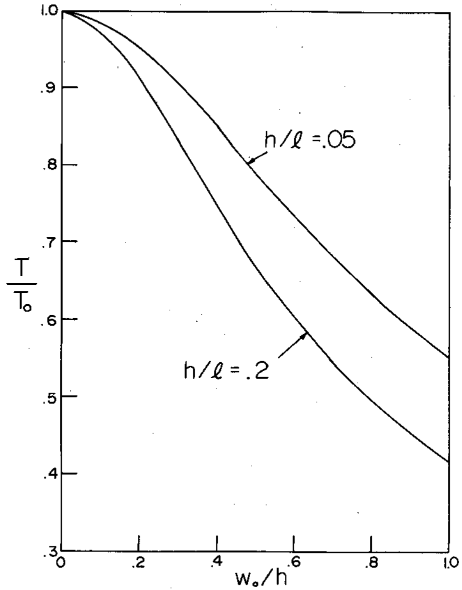

Fig. 3 Influence of large amplitude on the period of vibration of a hinged composite beam with T/T_o vs w_o/h.

in which

$$\beta^2 = [2(1 + 1/g)]^{-1} \quad (50)$$

$$\omega^* = \omega_0(1 + g)^{1/2} \quad (51)$$

where

$$g = \pi^2(2a_2 - a_1)w_0^2/8l^2(a_3 - a_4\psi^* - a_5\Phi^*) \quad (52)$$

$$\omega_0 = \pi[a_3 - a_4\psi^* - a_5\Phi^*)/b_1]^{1/2} \quad (53)$$

The vibration is periodic, with period T in time given by

$$T = 4K(\beta)/\omega^* \quad (54)$$

where $K(\beta)$ is the complete elliptic integral of the first kind. The period for the linear case T_0 can be obtained by setting $g = 0$ and $\beta = 0$. Noting that $K(0) = \pi/2$, we have

$$T_0 = 2\pi/\omega_0 \quad (55)$$

Thus, the ratio of periods is

$$T/T_0 = 2K(\beta)/\pi(1 + g)^{1/2} \quad (56)$$

This ratio is evaluated for the numerical parameters given by Eq. (37) as function of the dimensionless amplitude w_0/h. Two cases—$h/l = 0.05$ and $h/l = 0.2$—are presented in Fig. 3. In Fig. 4, the ratio of periods is plotted against w_0/l. It is observed that the period decreases substantially as the amplitude increases. Moreover, such a decrease is more pronounced in the case of thick beams.

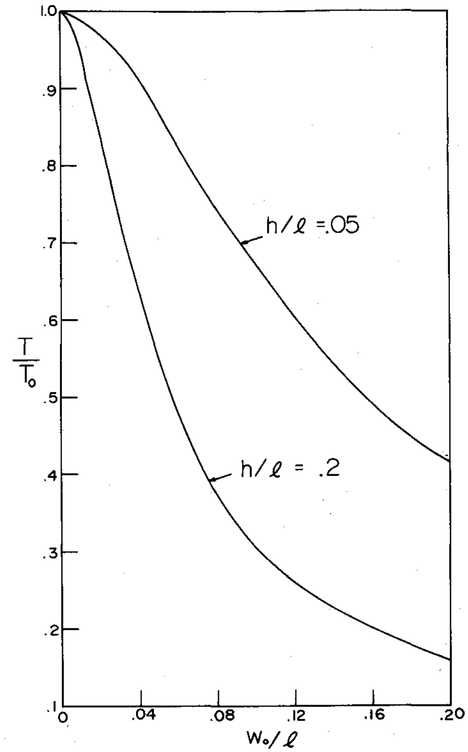

Fig. 4 Influence of large amplitude on the period of vibration of a hinged composite beam with T/T_o vs $w/_o/l$.

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